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Is there any connection between the network morphology and the fluctuations of the stock market index?

F.M. Stefan [a,*], A.P.F. Atman [b]

[a] Programa de Pós-Graduação em Modelagem Matemática e Computacional, Centro Federal de Educação Tecnológica de Minas Gerais - CEFET-MG, Brazil

[b] Departamento de Física e Matemática and National Institute of Science and Technology for Complex Systems, Centro Federal de Educação Tecnológica de Minas Gerais - CEFET-MG, Av. Amazonas 7675, 30510-000, Belo Horizonte-MG, Brazil

HIGHLIGHTS

- Behavioral model to study market index fluctuations in several scenarios.
- Investigation of different trust network morphologies on the index oscillations.
- Remarkable effects due to complex network synchronization in anti-imitator scenario.
- Fluctuations of the stock market index are heavily biased by the network morphology.
- Mixing scenario enhances self-affine features of the stock market index.

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ABSTRACT

Models which consider behavioral aspects of the investors have attracted increasing interest in the Finance and Econophysics literature in the last years. Different behavioral profiles (imitation, anti-imitation, indifference) were proposed for the investors, which take their decision based on their trust network (neighborhood). Results from agent-based models have shown that most of the features observed in actual stock market indices can be replicated in simulations. Here, we present a deeper investigation of an agent based model considering different network morphologies (regular, random, small-world) for the investors' trust network, in an attempt to answer the question raised in the title. We study the model by considering four scenarios for the investors and different initial conditions to analyze their influence in the stock market fluctuations. We have characterized the stationary limit for each scenario tested, focusing on the changes introduced when complex networks were used, and calculated the Hurst exponent in some cases. Simulations showed interesting results suggesting that the fluctuations of the stock market index are strongly affected by the network morphology, a remarkable result which we believe was never reported or predicted before.

1. Introduction

In the past decades, several interdisciplinary approaches led to the development of the field of Econophysics, which integrates knowledge from Economy, Statistical Physics, Computation, Mathematics, Sociology, and Psychology in order to quantitatively describe the actual behavior of the financial market [1–5]. Despite the intense debates in the literature opposing *graphists*, *fundamentalists*, *behaviorists* etc., which only express the residual reductionism from the specific fields of

* Corresponding author.
E-mail address: fischersmath@gmail.com (F.M. Stefan).

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the researchers, the use of statistical physics tools, as fractal analysis or Fokker–Planck equations, and computational techniques, as agent-based models or numerical integration of differential equations, has greatly improved our comprehension of how the real market works [6–11]. Of particular interest, the temporal series of the stock market index is widely studied by means of analytical and numerical analysis [2,12], both qualitatively and quantitatively, but, for the moment, they still resist to be tamed.

Stock market indices are affected by a lot of situations which can cause fluctuations. Since the value of the index depends on the predominant choice of the investors in a given moment [13], it can be affected by a wide range of situations such as financial reports from companies, news on the media, and even by simply following the choice of an investor's best friend [14]! The development of the modern Game Theory in the mid of Section XX, from the seminal work of John von Newman [15] has demonstrated the importance of this subject to several areas, from Economics to Biology, passing by Psychology, Mathematics etc. [16]. Of particular interest for the Finance literature, Minority Games (MG) [17], Cooperative Games, Zero-Sum Games and Behavioral Models play a major role on the description and analysis of real market systems [3,9,18–20].

In recent years, hundreds of papers have demonstrated that most part of the features of real markets can be quantitatively and qualitatively described by computational models using Game Theory ideas combined with Statistical Physics tools. A particular attention has been devoted to agent-based models, due to its great applicability in the study of fluctuations in financial markets [7], which exhibit fascinating statistical properties [3]. In these models, it is common to observe the agents working on the lattice, with a local neighborhood, and several instances are proposed for the investors dynamics. The investors act like a crowd–anticrowd MG [21,22] by taking into account their psychological behavior (imitation or anti-imitation), which is fundamental for more realistic market games [23–26]. Minority Games can differ in the type of microscopic dynamics used (e.g. stochastic versus deterministic), in the definition of the information provided to the agents (real-valued versus discrete), in the agents decision making strategies, and also in the specific recipe used for converting the observed external information into trading actions [27]. Thus, a huge range of possibilities are open to explore combining Game Theory ideas, Statistical Physics tools and computational modeling.

A particular situation occurs when the investor's decision is based only on the behavior of his trust network. In this case, psychological tendencies arise from various processes [28], for instance, by spreading the information away and by social influence. Thus, sometimes the investors act in a completely irrational way, weakening the basic premise of the efficient market hypothesis [19,29]. Even if this kind of situation is not observed all the time, it is essential to the development of the so-called crisis of the financial markets [1].

Real markets exhibit several phenomena which can be associated to this irrational behavior of the investors, causing avalanches – large fluctuations of the number of investors buying or selling stocks – which are reflected in large oscillations of the stock market index [9], a mark of a period of crisis [1]. In the last decades, the statistical physics community has turned its attention to the task of extracting information from these index fluctuations using new theoretical approaches based on fractal geometry and chaos theory and, very frequently, by applying computational modeling [3,8,30,31]. The most used tool to quantify the stock index time series is the Hurst exponent [32,33], in particular, the detrended fluctuation analysis (DFA) [34,35] is widely employed.

Another interdisciplinary field which has experienced an increasing interest in last years, is the development of the, so called, Complex Networks. This multidisciplinary subject integrates computation, graph theory, statistical mechanics, sociology and economics [36–38] enhancing the potential power for prediction of crisis by combining several tools to analyze the data, providing a plethora of computational techniques which make possible the studying of economical systems by means of simulations. Initially motivated by the expansion of the internet and the need of understanding the properties of the www network, this subject migrates from mathematics and computation to biology and sociology when the researchers discovered that several social networks, as the sexual or postal ones [38], exhibit the same power law features as they observe in ecological or metabolic networks of living organisms [39]. Besides, networks are essential for the spreading of information, and play a central role on social relations [40]. Recently, some works aimed to investigate the topology of trust economical networks [41–45] but there is no comprehensive study on this subject yet.

In this paper, we have extended an agent based model [31,46] to study the behavior of the investors in the stock market focusing on the relation between the trust network of the investors and the stock market fluctuations. We considered different network morphologies for the investors trust network, and tested the dependence on the initial conditions for the state of the investors in each situation. Four scenarios of behavioral profile of the investors were considered, and we have studied the temporal evolution of the stock market index which was obtained in each scenario. We show that, for some scenarios, different network morphologies lead to completely different behavior on the fluctuations of the stock market index, and some results were totally unexpected. The paper is structured as follows: firstly, the networks used along the work are characterized and the model is presented; secondly, we show the results for the temporal evolution of the stock market index in each scenario studied. Finally, we close with our conclusions and perspectives, followed by the references.

2. Methodology

In this section we describe the techniques and models used in simulations. The algorithms considered to build the investors' trust network are discussed in the next subsection. Then, in the following subsection, we present the behavioral agent based model detailing the cellular automaton rules used and the scenarios investigated.

2.1. Networks

In real markets, the morphology of the trust network of the investors cannot be easily assessed [41,43]. To our knowledge, at the moment it is not possible to investigate or even to understand whether or how the morphology of the network plays any role on the oscillations of real stock market indices. Here, we propose a methodology to investigate this problem by means of numerical simulations, using a cellular automata model in which is possible to test different morphologies for the investors' trust network, expecting to correlate them to specific features of stock market oscillations.

In order to characterize the networks that we have used, we present some statistical properties of the morphological parameters of the networks which were considered: regular, random conservative, nonconservative and complex. As stressed by M.E.J. Newman [40], the importance of studying social network phenomena by characterizing and modeling different kind of networks is the possibility to see how the information spreads in each network. Thus, *a priori*, we can expect that the network morphology should play a major role on the market dynamics.

The study of networks has increased considerably in the last years due to the remarkable advance of the computer performance and the wide spreading of the internet. Besides, the discovery that several social and biological networks exhibit the same universal characteristics has enhanced the interest of understanding the influence of the network on the dynamics of complex systems [37,38]. Initially proposed as an extension of Graph Theory, the study of networks can be traced to the original works of Erdös and Rényi [47] in the study of random graphs. Later, Barabási and Albert popularized the properties of the so-called Small World Networks (SWN) [48,49], which turned out to be ubiquitous in social and biological dynamical systems, and was characterized by a power law degree distribution [50].

The first network we have considered was a regular square lattice which served as the substrate where the trust network was built in. Each node represents an investor, and his trust network grows attaching links (connections) between the node and its neighborhood. In the *regular case*, the morphology considered for the trust network was the Moore neighborhood (eight cells). In regular lattices, all investors have the same number of neighbors, there is no preferential attachment and the network is static, in the sense that the neighborhood does not change on time (this last feature stands for all cases studied here). All links are bidirectional, *i.e.*, if the investor i is connected to investor j, the reciprocal relation is true, j is also connected with i.

For the *random networks*, we consider two implementations: a conservative case, where the number of neighbors is fixed to eight, but randomly distributed on the lattice; and a non-conservative case, where the number, and the position, of all connections were randomly sorted. We summarize the algorithms used to build the networks as follows:

- *Conservative case*: starting from a square regular lattice with N sites and Moore neighborhood ($8N$ links), we perform $4N$ random permutations on the regular connections. In such way that, all sites will have the same number of neighbors in the end, but in average half of its initial eight regular links will be shifted randomly along the lattice;
- *Non-conservative case*: starting with N sites in a regular lattice, we sort $8N$ connections among random pair of sites. In that way, in the end, each site will have a random number of neighbors, between 0 and $N-1$, randomly distributed along the network.

Note that for the random case, the bidirectional property is loosed, since some of the connections will be unidirectional. This feature is enhanced in the non-conservative case.

In the case of SWN, we consider the Barabàsi–Albert algorithm [48] to build scale free, or small world, networks. Basically, the code considers a preferential attachment of the links in such way that, the greater the number of links of a node (investor), the higher the probability of a new node to be connected to it: the rich gets richer! Thus, we are able to generate SWN up to N nodes and $8N$ links, whose distribution by node follows a power law, $\mathcal{N}(\ell) \sim \ell^{\gamma}$. The exponent measured, $\gamma \sim -2.5$, agrees with the expected value for the Barabàsi–Albert model for networks of comparable sizes.

2.2. Behavioral model for investors

The model that we consider here is an extension of an agent based model [31,46] originally proposed to take into account the behavioral profiles of the investors to study their influence on the stock market index. The original model has employed an agent based algorithm, using a hybrid cellular automata model and Monte Carlo method, making it possible to reproduce the complexity observed in real markets for some specific scenarios [46].

Basically, the model considers a cellular automaton defined on the networks presented in the preceding subsection: each node represents an investor and different neighborhoods correspond to the investor's trust network. The investors act directly on the stock market and can take three different decisions, represented by the states **buying** (B), **holding** (H) or **selling** (S) stocks. Note that the investors negotiate directly with the stock change, and do not make operations with each other. The investor decision is strongly influenced by the trust network. Besides, each investor has a given behavioral profile among 3 possibilities:

- *Imitation*—the investor imitates the state of the majority of the investors in his trust network;
- *Anti-imitation*—the investor imitates the state of the minority of the investors in his trust network;
- *Indifference*—the investor choices are taken at random and do not depend on the state of his trust network.

The trust network of the investors is made up of the neighborhood of the corresponding node in each morphology considered. At each time step, the investor considers the decisions made by his trust network and, depending on his behavioral profile, he takes an option, either imitating or anti-imitating the majority option performed in the trust network. The investor can even make a random choice, that means that he is totally indifferent of what his neighborhood is performing at the preceding time step.

We have tested some different initial conditions in order to verify how the dynamic of the model is biased due to particular initial configuration, in each behavioral profile scenario. Considering the square lattice used as substrate for the node attachment, we tested three different initial conditions:

Random—the initial state of the investor at node *i* is randomly sorted among the three possibilities (H, B, S) defined above, with equal probabilities to each one of them; *Alternate*—the initial state depends on the parity of the column in which the node is placed in the lattice, filled alternating the states B, S or H successively. *Striped*—the substrate lattice is divided in three regions – stripes – and the initial state of the investor follow that of the region in which he is located.

Once defined the initial state of all investors, the algorithm sets up the behavioral scenario chosen to the investors. We test four scenarios:

- A—all investors are imitators;
- B—all investors are anti-imitators;
- C—all investors are indifferent;
- D—one third of the investors are imitators, one third anti-imitators and one third indifferent.

The psychological profiles of the investors are static, in the sense that they do not change with time, and when there is no clear majority (minority) in the trust network, the algorithm randomly sort between the two majority (or minority) options depending on the investor profile, except in the case that the investor has no neighborhood at all. In this case, the investor will keep his preceding option—a *stubborn*.

The stock market index was initially set to 100 and updated at each time step considering the net balance between the number of investors buying and selling stocks, divided by the system size. We have considered initially the case of unlimited amount of resources (stocks and money) – Fig. 3 – in order to validate the model, and then we have imposed limits to the quantity of money and stocks. Basically, the investors can sell (buy) stocks only if he has stocks (money) sufficient to perform the action. Otherwise, he holds the stocks. Thus, the parameters analyzed during simulations were the amount of money and the number of stocks of each investor in each time step. All results were obtained with constrained resources, except when explicitly indicated. The simulations ran until a stationary pattern was observed of the stock index. Thus, by following this procedure, we were able to highlight the changes introduced when different trust networks morphologies were used as trust neighborhood, an attempt to isolate the influence due only to the network morphology.

Fig. 1 shows a comparison of the stationary states obtained from two different initial conditions, at the scenario B (all anti-imitators). We have observed that only the scenario A exhibits a dependence on the initial conditions. This result could be anticipated since that this scenario (all imitators) corresponds to a strong local correlations, and every sample should evolve towards a frozen configuration which depends on the particular initial state. We believe that this scenario could be related to Ising, or Potts, universality class, since it is analogous to a zero temperature three state Ising model. For all other situations, scenarios B, C and D, no clear dependence on the initial conditions was observed.

3. Results

In this section we show detailed results for the scenarios described above. For each set of parameters, 10–20 samples were averaged to estimate the values for the exponents and build the distributions. The calculation of the Hurst exponent follows the detrended fluctuation analysis [34,35], and was performed only in the cases where the index exhibits self affine properties. We present the results separately for each scenario tested.

3.1. Scenario of all indifferent

The first scenario considered was the case when all investors take their decisions independently to their neighborhood. This behavioral profile is called *indifferent*. This scenario should lead to trivial results and is useful to validate the model. As the investors decisions are taken randomly, the assumptions for the Efficient Market Hypothesis are all valid. The prices of the stocks are not under any tendency from the trust network, once the decision of the investors does not take into account the state of their neighborhood. Thus, the fluctuations of the stock market should behave analogously to a random walk, with no development of correlations in the stock index. Therefore, we can expect a Hurst exponent of 1/2 for this case. Besides, neither dependence on the network morphology, nor due different initial conditions are expected to happen in this scenario. In Fig. 2 it is shown that all these predicted behaviors were confirmed by simulations. The average Hurst exponent measured at this scenario agrees with the expected value, considering finite size effects— Fig. 4 (left side).

We have verified that the morphology is not crucial in this scenario and the initial conditions do not influence the dynamic of the market. Both results were expected since the decision of the investors is made completely at random and does not depend on the neighborhood in this scenario. Moreover, we can verify the stochastic feature of the model by observing the characteristic behavior of the stock market index expressed in each case and quantified by the Hurst exponent. The measured values were: Regular: $H = 0.470 \pm 0.001$; Conservative: $H = 0.4560 \pm 0.0015$; Random $H = 0.496 \pm 0.003$;

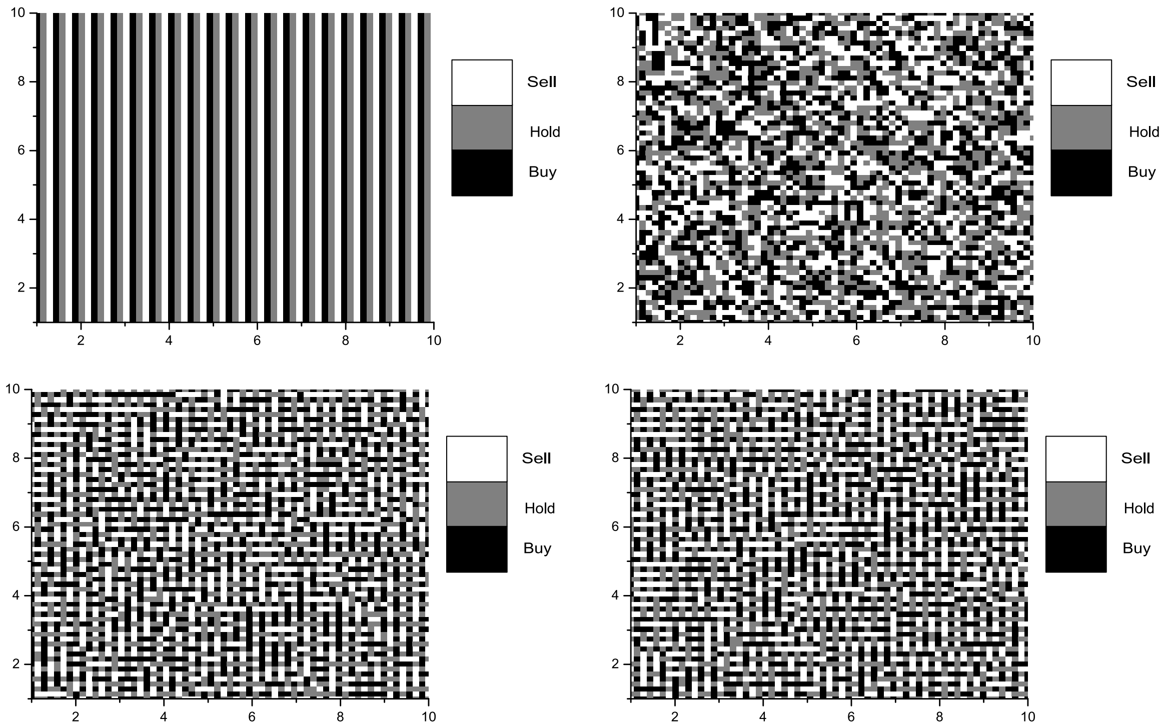

Fig. 1. Evolution of the state of the investors in function of the initial conditions. The scenario considered was 100% of anti-imitators and we showed on the upper panels the initial conditions: alternate at left and random at right. The bottom panels show the stationary state corresponding to each initial condition. It is clear that, for this scenario, the initial condition does not play any role on the dynamics of the model.

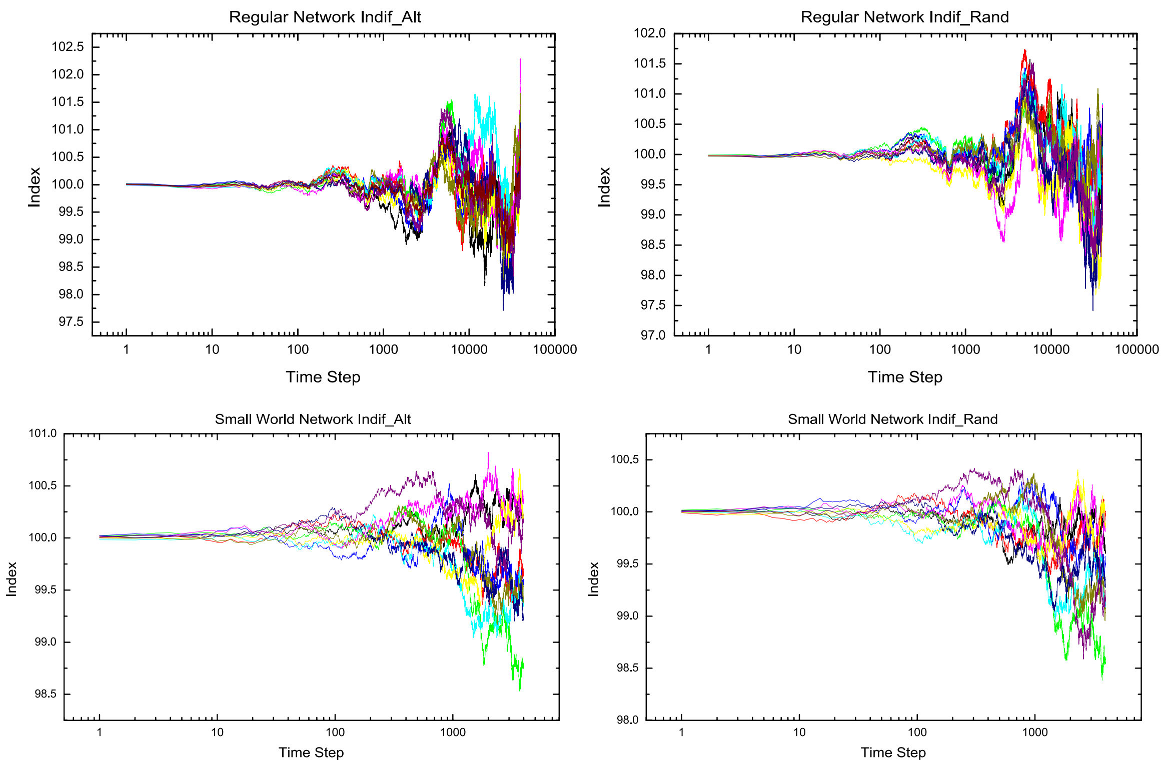

Fig. 2. Scenario on which all investors are Indifferent. The upper panels consider regular lattice as trust network, and the bottom panels consider SWN lattices. We show that different initial condition does not influence the model dynamics for this scenario. Initial Conditions: left panels—alternate; right panels—random.

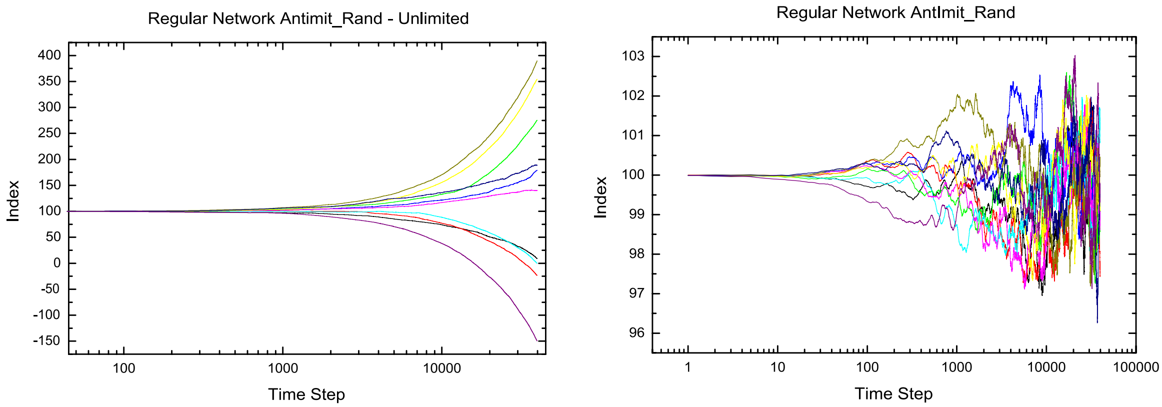

Fig. 3. Effect of limiting resources. Left: unlimited resources case. Right: limited resources case. A scenario of all anti-imitators was considered. Note the exponential feature of the index to the unlimited case, growing or decaying depending on the random initial conditions. In the limited case, the index evolves to a stationary state due to the limitation of resources (note the scale in the y-axis). The different colors correspond to different simulation runs, using different seeds for the random number generator. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

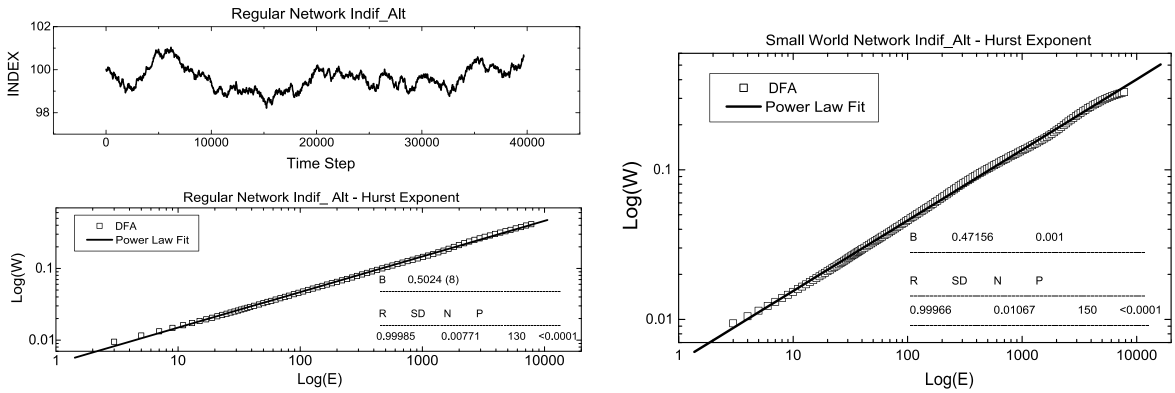

Fig. 4. The Hurst Exponent was calculated by applying DFA (Detrended Fluctuation Analysis) for the two scenarios. The one of all indifferent investors (left) having a regular network as trust network, alternate initial condition has the Hurst exponent $H = 0.5024 \pm 0.0001$. The other one (right) Hurst exponent is $H = 0.497 \pm 0.002$ for the same scenario, but using SWN as the trust network.

Small World: $H = 0.4759 \pm 0.0008$. It is clear that there is no tendency on the measured exponents, and all agree with the efficient market hypothesis.

3.2. Scenario of all imitators

In this scenario the investors are all imitators, taking the same option as the majority of their trust network. When it happens to have two or more predominant options, the algorithm randomly chooses one of them. This scenario corresponds to a high correlated system with strong local interactions. We can anticipate that the system should evolve to an absorbing state corresponding to the minimum "energy" configuration for a given initial condition and network morphology. Fig. 5 shows the stock market index evolution for different network morphologies and initial conditions. The first remarkable feature is that, for all examples displayed, the stock index reaches a stationary state, which is frozen, corresponding to an absorbing state. We can observe that there are many of these absorbing states. Note that, for some samples in the upper panels, the stock index initially grows with time, indicating that there are more investors buying than selling. It is compatible to a scenario of competition of two clusters of investors buying and selling; as greater is the difference between the sizes of the clusters, faster is the growing/decaying of the stock index. Suppose, for instance, that this difference between selling and buying investors is small, as it can be observed in the curve at the center of the upper left panel in Fig. 5. We can see that initially the index grows, expressing the fact that the buying cluster is greater; around the time step 90, the money available to buy stocks vanishes, and the selling option becomes predominant; thus, the index starts going down until all the stocks are sold, and the system gets frozen at the holding state. An analogous, but opposed situation, occurs in the plots where the index initially decreases: before freezing, the index inverts the tendency for some few steps expressing the residual activity of investors buying stocks.

A remarkable and not anticipated result was observed when complex networks were employed in this scenario. In the bottom panels of Fig. 5, we observe that the results display a remarkable convergence. Note that the same set of seeds for the

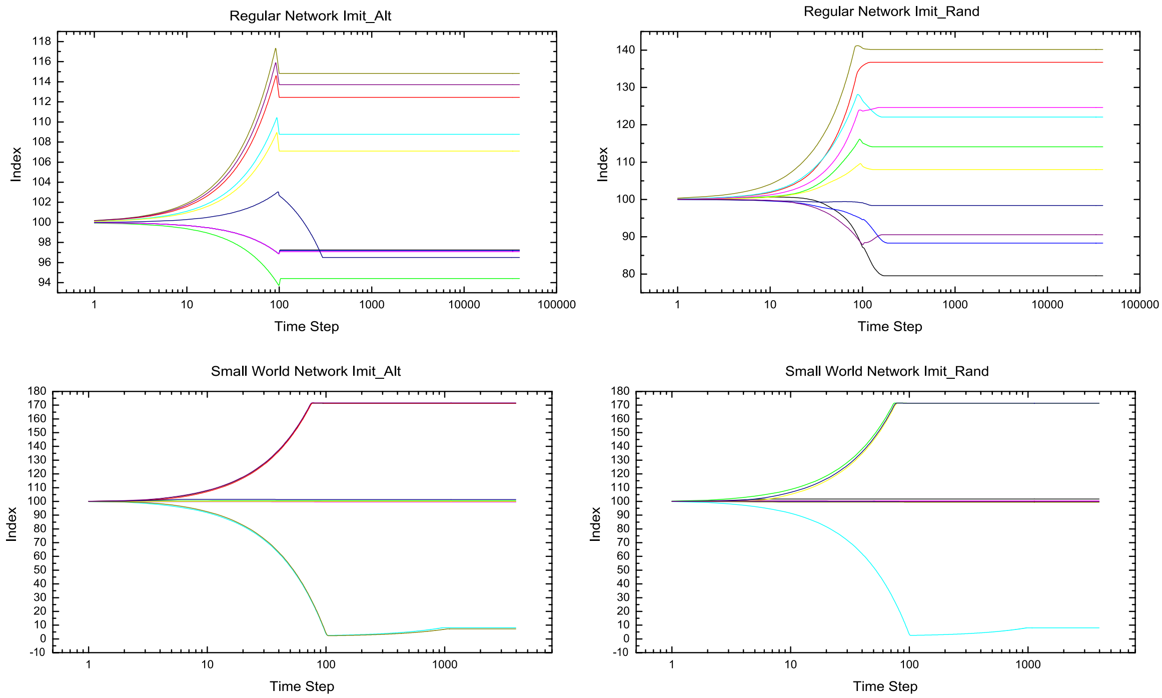

Fig. 5. The scenario of all Imitators having two different initial conditions. In this case, the initial conditions influence the stock market index, and the dynamic of the model is altered due to the initial conditions. Upper panels: left – Regular Network – alternate case; right – Regular Network – random case. Bottom panels: left – Complex Network – alternate case; right – Complex Network – random case.

random number generator was employed for each morphology and in each case analyzed. We believe that this radical change in the index evolution is due to the characteristic distribution of links verified in the complex networks: a single investor with a large number of links (a "hub") can influence regions which spans along all network, strongly biasing the index, introducing synchronization on the dynamics of the model. This effect was previously observed when complex networks were used in different models [51,52]. Note that some samples evolve to a growing stock index, indicating a preference to buy stocks, while others evolve to a crash in the market, and sometimes the index remains unaltered, indicating an equilibrium between selling and buying options. In this scenario, the profiles did not display self-affine features, and thus, we did not calculate the Hurst exponent.

3.3. *Scenario of all anti-imitators*

In this scenario, the investors decision is opposed to the majority observed in their trust network; thus, they anti-imitate the preferential option of their neighborhood. When the preferential option is to hold, the algorithm sorts randomly between selling or buying options. This anti-imitation feature inserts a frustration mechanism which avoids the system to reach a frozen state, opposed to the situation verified in the preceding scenario. We exemplify this feature showing the dependence of limiting resources (quantity of money and quantity of stocks) in the model. In Fig. 3, we show the behavior of the stock index for 10 samples enhancing the effect caused by imposing limitation of the resources in the system. Note that, even in the case of limited resources, when the samples reach a stationary state, it does not freeze and the index still keeps fluctuating.

Again, when complex networks are considered, the results display some remarkable changes as observed in Fig. 6. We observe for the cases of Regular and Random networks results showing randomness and anti-persistence typical of the frustration dynamics from anti-imitating scenario. The higher is the randomness of the system, the greater is the anti-persistent behavior of the index, but the range of the values is significantly reduced— Fig. 6. This feature is compatible with the increasing of unidirectional links as the randomness increases in the network, indicating a positive correlation between these features.

However, when Small World networks are used, with the same set of random seeds, it is easy to see that the stock market index displays a persistent feature, only showing anti-persistence over larger scales—above 100 time steps. These transients of a persistent behavior before inverting the tendency are indications of the avalanche behavior, typical of real markets. Note that in the case of SWN networks, due to the algorithm for building the network, all links are bidirectional, as in the regular case, only the link distribution is completely different.

We believe that we can explain this odd behavior considering that the "hubs" in this scenario, assume positions opposed to the majority of investors, mining the efficiency of the frustration mechanism. Imagine the following situation which illustrates an extreme case of SWN: suppose a network with 100 investors where, let us say, 5 investors are hubs connected

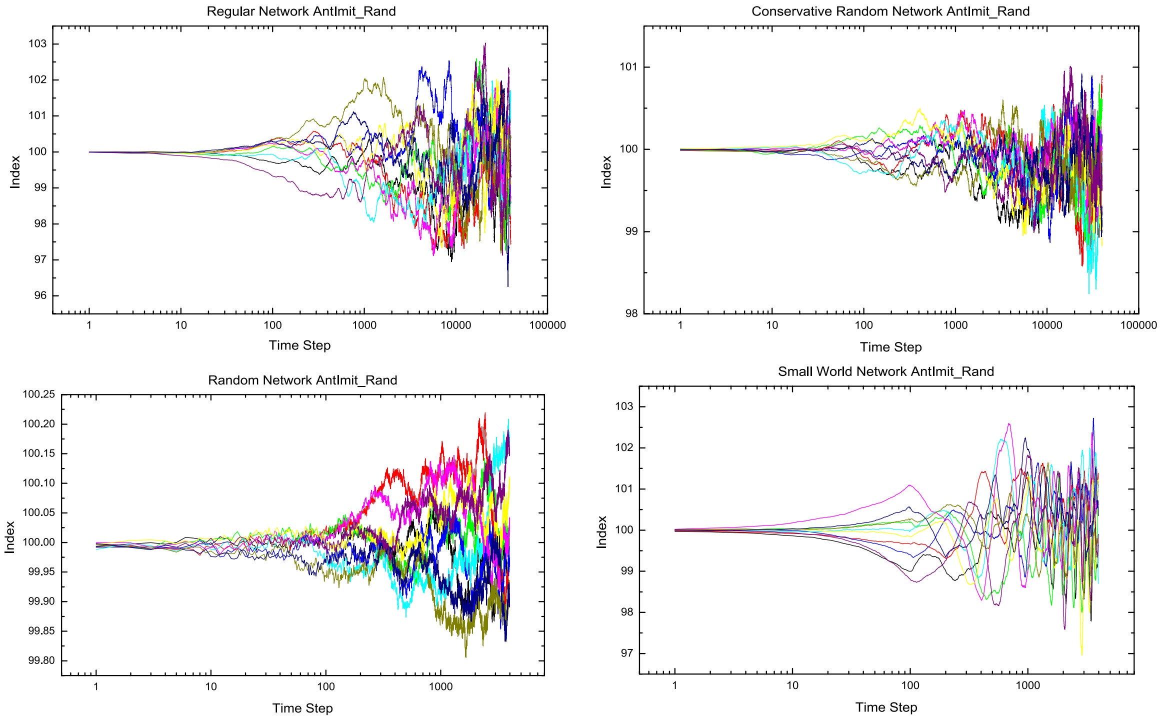

Fig. 6. The scenario of all Anti – Imitators having the same initial condition – Random. The morphology has a strong influence over dynamics. Upper panels: left—Regular Network; right—Conservative Network. Bottom panels: left—Random Network; right—Small World Network. Note the range of the ordinate in each panel. When randomness is added to the neighborhood, the index fluctuates more, but the range is very reduced. However, for SWN networks, the frustration mechanisms fails, and a quite persistent behavior is observed, with anti-persistence only at large scales.

to 50 or more investors, and all the other 95 investors are connected up to 3 other investors only ("hubs"). It is easy to realize that if the hubs decide, for example, to buy, it is because the majority of the investors are selling. Besides, the investors which are linked to the hubs will anti-imitate them, so they continue to sell the stocks, exhibiting a high degree of synchronization. As consequence, the index does not fluctuate, since the frustration mechanism does not work anymore, and will decrease in a persistent way until the stocks vanish. Then, an inversion of the tendency is observed, with the hubs selling and the majority buying, and the index grows until the money vanishes. This alternate mechanism makes the index show persistence at short scales and anti-persistence over large scales.

This example is particularly enlightening to understand how the frustration feature of the anti-imitation strongly depends on the network morphology, or even to initial conditions. It is worth to mention an important observation made in the Regular network using random and alternate initial conditions in this scenario: for the random case, the index behaves as expected, fluctuating randomly for different sample seeds, but when the alternate initial condition is used, the index remains frozen until the hundredth time step when, due to resources limitation, the dynamics is altered to a transient and stationary states.

The singular dependence of the oscillations of the market index in this scenario is confirmed by the Hurst exponent measured in each network morphology. In Fig. 7 we show the results for the mean roughness in function of the scale, for anti-imitation scenario. Clearly, we observe that the curves displays different features in each morphology. In regular networks, the curve has two regimes: first, a persistent behavior with $H > 0.5$ until $\epsilon \sim 100$ time steps, and then evolves to a $H \sim 0.5$ limit, close to a random walk. As we increase the randomness of the network, the inclination of the plot decreases, as we observe for Conservative $H \sim 0.5$, and Random $H \sim 0.25$ cases, enhancing the anti-persistent behavior. However, for SWN network, a distinct behavior was observed: first a very persistent regime in small scales, with $H > 1$, and then a transition to a strong anti-persistent regime, $H \sim 0.15$, for larger scales.

In our opinion, these both features are very important to understand the role of the anti-imitation profile in the model, and also to better evaluate how the frustration mechanism can be avoided or reinforced with complex morphologies, when long-range correlation between the neighborhood and unidirectional links are present. This point deserves a more detailed analysis which is beyond the scope of this paper, and we intend to explore it in further investigations.

3.4. Scenario of mixing

The most interesting scenario considered in our study was when we mixed the three behavioral profiles equally among the investors, with 1/3 of investors of each profile: imitation, anti-imitation and indifferent. We show in Fig. 8 the stock

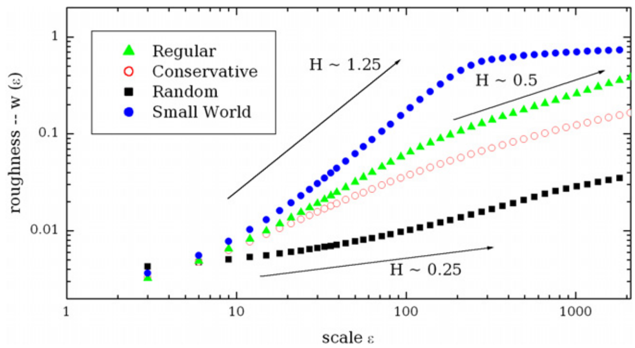

Fig. 7. The dynamic is under a strong influence from network morphology. The scenario considered is of all anti-imitators and random initial conditions. The dependence of the roughness with the scales is shown to each morphology considered.

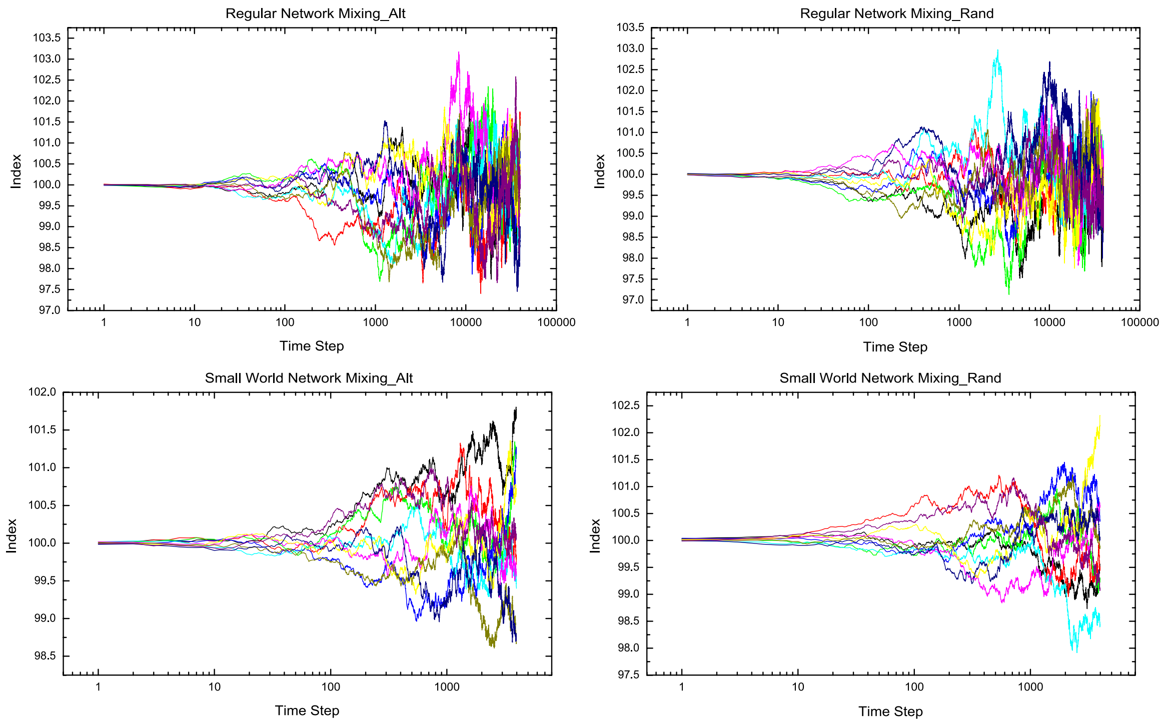

Fig. 8. The scenario considered is of mixing (1/3 of Imitators, 1/3 of Anti Imitators and 1/3 of Indifferent) having two different initial conditions—random and alternate. Upper panels: left: Regular Network—Alternate. Right: Regular Network—Random. Bottom panels: left: Small World Network—Alternate; right: SWN Network—Random.

index evolution for the different network morphologies considered. Here, surprisingly, we do not notice any remarkable difference in the index fluctuations with network morphology at first glance. However, analyzing the values of the Hurst exponents measured in each case, we notice that there is a kind of dependence of the Hurst exponent on the randomness of the network: Regular $H = 0.621 \pm 0.008$; Random conservative: $H = 0.506 \pm 0.012$; Random non-conservative: $H = 0.475 \pm 0.028$; Small World: $H = 0.53 \pm 0.03$. It is interesting to notice that the SWN morphology displays Hurst exponent larger than the Random cases, maybe an indication that the synchronization effect on SWN networks can act as a positive correlation. Another conclusion analyzing the results is that the higher is the organization of the lattice, more persistent is the index on this scenario, a remarkable feature due only to the trust network morphology.

It is worth to mention that the stationary state which exhibits self-affine characteristics compatible with actual markets, an observation already noted by Ref. [46]. An additional study which we performed was to consider the case of low density of links in complex morphologies. We show the results in Fig. 9. It is worth to note that, in this case, the "stubborn" profile, investors with no trust network, will play a major role in the index evolution, as we can observe in this figure. We can notice that an initial bias is observed in the index due to the balance between selling and buying option on the stubborn investors. Around time step 100, the stock (money) of the stubborn vanishes, and then it does not influence the index anymore, which behaves differently depending on the scenario considered.

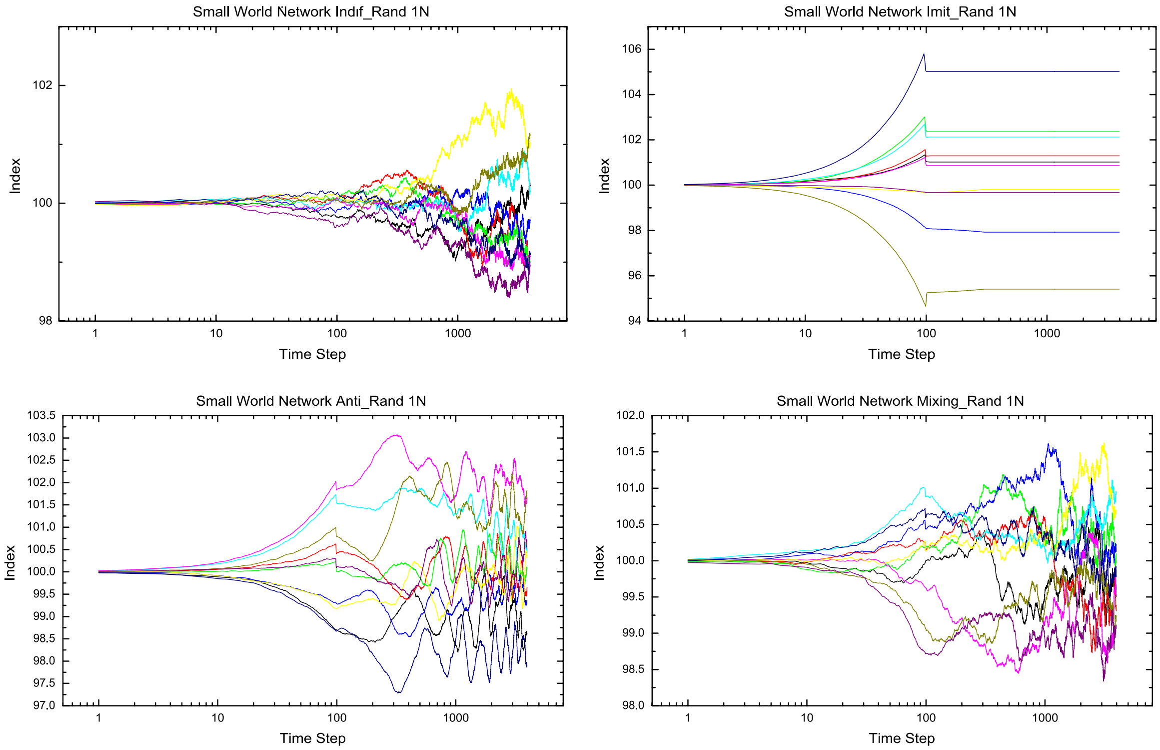

Fig. 9. Study of the limit of low density of links. On these panels we consider SWN networks with N links distributed on a power law. The presence of investors without neighborhood is enhanced in this case—stubborns. We show the results for all four scenarios considered: indifferent—top left; imitators—top right; anti-imitators—bottom left; mixing—bottom right.

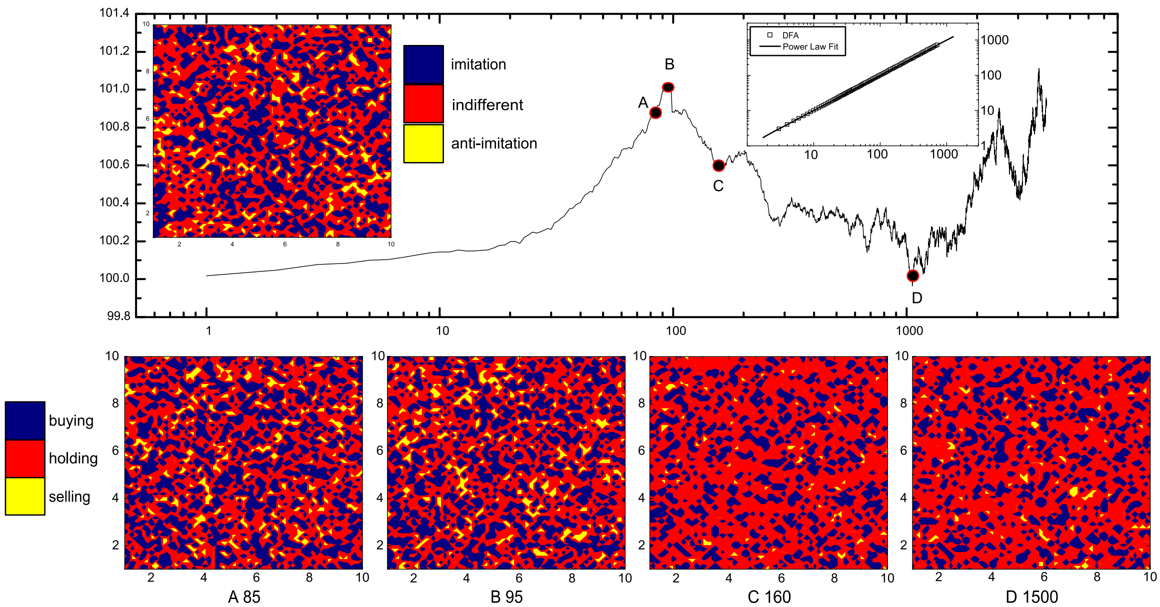

Fig. 10. Detailed analysis of the model for the case of mixing scenario for SWN network with low density. We show the state of the system at four different moments, as indicated in the main panel by the letters A, B, C and D. For each point, we plot in the bottom panels the state of each investor of the network, with a color legend. We also show the behavioral profile of the investors in the top left inset, and the Hurst exponent measure in the stationary phase—top right inset. We measured $H = 0.60 \pm 0.01$ for this sample. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

We present a detailed study of this transient in Fig. 10, where we show snapshots of the system at different time steps during its evolution. First, we clearly observe that the initial evolution is dominated by the stubborn, which leads the index to a consistent growing; after some time steps, the available money for the investors finishes, and the index decays fast due to the residual activity of selling stubborns. The system, then, encounters a stationary state which exhibits strong self-affine characteristics. We believe that this feature in the stock market indices reflects the competition of two major driving forces in the system: frustration, from anti-imitation behavior, and synchronization, due the combination of imitation and the presence of hubs in the networks. In Fig. 9 we can also observe the oscillations pattern of the index from each behavioral profile separately, and it is interesting to note that, in the mixing case, the index seems to incorporate all three different pattern in its behavior!

4. Conclusion

In this paper we have extended a cellular automata model of investors' behavior in the stock market to consider different morphologies for the trust neighborhood of the investors. We show that the trust network morphology can alter drastically the dynamic of the model in some scenarios. We have simulated several scenarios which are driven by the psychological profile of the investors (imitators, anti-imitators and indifferent) and by the network morphology—synchronization. We have studied the influence of the trust network morphologies in the stock index oscillations and have observed that small world networks lead to unexpected results when they are compared to the regular and random cases. Simulations results have shown that there is a connection between the network morphology and the stock market index oscillations, indicating that the investors' trust network morphology can determine the behavior of the stock market index, altering the dynamic of the fluctuations in some cases. To our knowledge, this result has never been reported before and was not anticipated by any study concerning the stock market index temporal series.

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